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Giant magnons in the D1-D5 system

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Abstract: We study giant magnons in the the D1-D5 system from both the boundary CFT and as classical solutions of the string sigma model in $AdS_3 \times S^3 \times T^4$. Re-examining earlier studies of the symmetric product conformal field theory we argue that giant magnons in the symmetric product are BPS states in a centrally extended $\mathrm{SU}(1|1) \times \mathrm{SU}(1|1)$ superalgebra with two more additional central charges. The magnons carry these additional central charges locally but globally they vanish. Using a spin chain description of these magnons and the extended superalgebra we show that these magnons obey a dispersion relation which is periodic in momentum. We then identify these states on the string theory side and show that here too they are BPS in the same centrally extended algebra and obey the same dispersion relation which is periodic in momentum. This dispersion relation arises as the BPS condition for the extended algebra and is similar to that of magnons in $\mathcal{N} = 4$ Yang-Mills

Keywords: D-branes, AdS-CFT Correspondence.

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Contents

1. Introduction

The duality between $\mathcal{N}=4$ Yang-Mills and string theory on $AdS_5 \times S^5$ is by far the most well studied example of the Maldacena correspondence [1, 2]. Another well studied and interesting example of the correspondence is the case of the duality between type IIB string theory on $AdS_3 \times S^3 \times T^4$ and the $\mathcal{N}=(4,4)$ superconformal field theory on a resolution of the symmetric product [1, 3, 4]

$$\mathcal{M} = T^N/S(N). \tag{1.1}$$

$AdS_3 \times S^3 \times T^4$ arises as a near horizon limit of the system of Q_1 D1-branes and Q_5 D5-branes wrapped on T^4, then N in (1.1) is given by Q_1Q_5. The duality sates that the spectrum of operators in the $\mathcal{N}=(4,4)$ superconformal field theory on $\mathcal{M}$ should be the same as the spectrum of type IIB string states in $AdS_3 \times S^3 \times T^4$. Operators which have large charges in the CFT should be dual to classical string configurations [5].

In this paper we consider operators with large J charges, here $J = J^3 + \tilde{J}^3$ the sum of the left and right SU(2) R-charges of the $\mathcal{N}=(4,4)$ conformal field theory. We study states with finite $\Delta - J$, where $\Delta = L_0 + \tilde{L}_0$ the left and right conformal weights of the operators. Operators with $\Delta - J = 0$ are chiral primaries which are the ground states of the $\mathbb{Z}_J$ twisted sector with J^3 charges $(\frac{J-1}{2}, \frac{J-1}{2})$. We can then consider finite number of excitations of the following form on the chiral primary

$$J^-_{p_1} J^-_{p_1} \cdots J^-_{p_j}|0\rangle \otimes |0\rangle, \tag{1.2}$$

the vacuum in (1.2) denotes the $\mathbb{Z}_J$ twisted sector. J^-_p are operators which lower the left J^3 quantum number and carry momentum p in the $\mathbb{Z}_J$ twisted sector, under the action of an element of $\mathbb{Z}_J$, $J^-_p|0\rangle$ picks up a phase proportional to integer multiples of p. We can therefore think of the insertions of J^-_p as magnons or impurities that move with momentum p

States of the form given in (1.2) were studied earlier in the limit of small momentum p and in first order in the $\mathbb{Z}_2$ blow up mode [6–8]. The dispersion relation of a single magnon was shown to be

$$\Delta - J = 1 + \frac{1}{2\pi^2}\lambda^2(Q_1Q_5)\frac{p^2}{4}, \tag{1.3}$$

where λ is the coupling of the relevant $\mathbb{Z}_2$ blow up mode in the symmetric product. In this paper we are interested in studying the magnons in the "giant magnon" limit given by

$$\begin{aligned} &J \to \infty, \qquad && \tilde{\lambda} = \lambda^2(Q_1Q_5) = \text{fixed}, \\ &p = \text{fixed}, && \Delta - J = \text{fixed}. \end{aligned} \tag{1.4}$$

This differs from the plane wave limit [9] where $\tilde{\lambda}$ is infinite and it is $n = pJ$ which is kept fixed.

Examining earlier studies of the magnons within perturbation theory in λ and the plane wave limit we argue that the magnons are BPS states in a centrally extended $SU(1|1) \times SU(1|1)$ superalgebra, the extended algebra has 2 more additional central charges. The centrally extended algebra can be written as a $\mathcal{N} = 2$ Poincaré superalgebra in 3-dimensions with a single central charge. The remaining central charges play the role of the 3-momentum in the Poincaré superalgebra. We then construct a dynamic spin chain representation of the extended algebra which carries these additional central charges and derive the following dispersion relation for a single magnon with momentum p

$$\Delta - J = \sqrt{1 + f(\tilde{\lambda})\sin^2\frac{p}{2}}. \tag{1.5}$$

where $f(\tilde{\lambda})$ is an undetermined function of the coupling $\tilde{\lambda}$. On rewriting the extended $SU(1|1) \times SU(1|1)$ as a $2+1$ Poincaré superalgebra the above dispersion relation can be viewed as the relativistic dispersion relation of a massive BPS particle in the $2+1$ Poincaré superalgebra. The spin chain representation constructed is such that these additional central charges vanish on physical states when we impose the momentum constraint, the algebra then collapses to the usual algebra. From the perturbative result in (1.3) we see that

$$f(\tilde{\lambda}) = \frac{\lambda^2 Q_1 Q_5}{\pi^2}, \quad \text{for} \quad \tilde{\lambda} \ll 1. \tag{1.6}$$

Equation (1.5) is a BPS relation and the magnons in question have large J charge, therefore we should expect to the find them as classical solutions to the string sigma model

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on $AdS_3 \times S^3 \times T^4$. These solutions are identical to the giant-magnon solutions found by [10] in $AdS_5 \times S^5$. Since these solutions only require the subspace $R \times S^2$, they continue to be solutions[1] in $AdS_3 \times S^3 \times T^4$ with Ramond-Ramond flux through the S^3. After the identification of the momentum of the magnons to the geometrical angle [10] in the classical solution we obtain the following dispersion relation for the magnon

$$\Delta - J = \frac{R^2}{\pi\alpha'}\left|\sin\frac{p}{2}\right|, \tag{1.7}$$

where R^2 is the radius of S^3 given by $R^2/\alpha' = g_6\sqrt{Q_1 Q_1}$ and g_6 is the $6d$ string coupling. Following the logic of [10] we write the giant-magnon solution in Lin-Lunin-Maldacena (LLM) [12] like coordinates for $AdS_3 \times S^3$ [13] and construct the Killing spinors of the geometry. From the solution of the Killing spinors and the stretched string like nature of the giant magnon in the LLM geometry we infer that the solution carries the required additional central charges to render it BPS in the extended SU(1|1) $\times$ SU(1|1) algebra. The BPS condition then implies the following dispersion relation at strong coupling for a single magnon

$$\Delta - J = \sqrt{1 + g_6^2\frac{Q_1 Q_5}{\pi^2}\sin^2\frac{p}{2}}. \tag{1.8}$$

Comparison with (1.5) we see that the

$$f(\tilde{\lambda}) = \frac{g_6^2 Q_1 Q_5}{\pi^2}, \qquad \text{for} \qquad \tilde{\lambda} \gg 1 \tag{1.9}$$

Thus identifying the coupling constant $\lambda = g_6$ and examining the weak coupling result in (1.6), perhaps we can guess that

$$f(\tilde{\lambda}) = \frac{g_6^2 Q_1 Q_5}{\pi^2}, \tag{1.10}$$

at all values of the coupling $\tilde{\lambda}$. Note that the dispersion relation in (1.5) also agrees with the plane wave limit when the equality in (1.10) is satisfied [6−9] The dispersion relation is similar to that of giant magnons in $\mathcal{N} = 4$ Yang-Mills with R^2 in the dispersion relation replaced by the radius of S^5 instead of S^3.

The organization of the paper is as follows: In the next section we review the results of the analysis of magnons at small p and small λ in the symmetric product pointing out the evidence for the extended SU(1|1) $\times$ SU(1|1) algebra. In section 3. we write down the extended SU(1|1) $\times$ SU(1|1) algebra and show that it can be written as a $\mathcal{N} = 2$ Poincaré algebra, we then construct a dynamic spin chain representation of magnons using this algebra which obeys the dispersion relation (1.5) and show that it is a BPS relation of the extended algebra. In section 4. we examine the magnons at strong coupling using LLM coordinates for $AdS_3 \times S^3$. We show that the magnons carry the required central charges to be BPS in the extended SU(1|1) $\times$ SU(1|1) algebra. appendix A, B fill in the details necessary to show that the giant magnon solution is supersymmetric. The method developed in appendix A enables one to determine the supersymmetries of of a solution of IIB gravity with $S^1 \times S^1 \times T^4$ isometry by embedding it as a solution of $(1,0)$ 6d gravity.

[1]Recently giant magnons in $AdS_3 \times S^3$ and related solutions were studied in [11].

2. Magnons in the symmetric product

In this section we present a short review of the symmetric product conformal field theory. We then specify the magnon excitations in the symmetric product whose conformal dimensions will be the subject of our interest and review the results of perturbation theory in λ.

The boundary theory corresponding to the system of Q_1 number of D1-branes and Q_5 number of D5-branes in type IIB on T^4 is given by the $\mathcal{N} = (4,4)$ super conformal field theory on a resolution of symmetric product orbifold

$$\mathcal{M} = (T^4)^{Q_1 Q_5}/S(Q_1 Q_5). \tag{2.1}$$

The global part of the $\mathcal{N} = (4,4)$ algebra is given by the supergroup $\mathrm{SU}(1,1|2) \times \mathrm{SU}(1,1|2)$. The two copies arise from the left movers and the right movers of the conformal field theory on $\mathcal{M}$. The bosonic part of the the supergroup $\mathrm{SU}(1,1|2)$ consists of the global part of the conformal algebra $\mathrm{SL}(2,R)$ whose generators are $L_0, L_\pm$ and the global part of the R-symmetry SU(2) whose generators are $J^3, J^\pm$. The 8 supercharges for $\mathrm{SU}(1,1|2)$ are labeled by $G^{ab}_{1/2}, G^{ab}_{-1/2}$, where $a \in \{+,-\}$ denotes the quantum numbers of the charges under $\mathrm{SU}(2)_R$ and $b \in \{+,-\}$ denotes the quantum numbers of the charges under $\mathrm{SU}(2)_I$ which is an outer automorphism of the $\mathcal{N} = (4,4)$ algebra. The subscript $\pm 1/2$ refer to the weights of the charges with respect to L_0. For our discussion the anti-commutation relations of relevance are

$$\left\{G^{++}_{-1/2}, G^{--}_{1/2}\right\} = 2(L_0 - J^3), \qquad \left\{G^{-+}_{1/2}, G^{+-}_{-1/2}\right\} = 2(L_0 - J^3). \tag{2.2}$$

From the above anti-commutation relations it is easy to see that the set of generators $\left\{G^{++}_{-1/2}, G^{--}_{1/2}, L_0, J^3\right\}$ or the set $\left\{G^{-+}_{1/2}, G^{+-}_{-1/2}, L_0, J^3\right\}$ each form a $\mathrm{SU}(1|1)$ sub-algebra with central charge $L_0 - J^3$. Similarly there is an identical copy of the $\mathrm{SU}(1,1|2)$ algebra from the right movers. We refer to these generators with a $\tilde{}$ superscript: $\left\{\tilde{L}_0, \tilde{L}_\pm, \tilde{J}^3, \tilde{J}_\pm, \tilde{G}^{ab}_{1/2}, \tilde{G}^{ab}_{-1/2}\right\}$. To be specific, and it will be justified by the subsequent discussion we will focus on the $\mathrm{SU}(1|1) \times \mathrm{SU}(1|1)$ subalgebra generated by the following

$$\left\{G^{+-}_{-1/2}, G^{-+}_{1/2}, (L_0 - J^3)\right\}, \qquad \left\{\tilde{G}^{++}_{-1/2}, \tilde{G}^{--}_{1/2}, (\tilde{L}_0 - \tilde{J}^3)\right\}. \tag{2.3}$$

The terms in the brackets $(L_0 - J^3)$ and $(\tilde{L}_0 - \tilde{J}^3)$ form the central charges of the $\mathrm{SU}(1|1) \times \mathrm{SU}(1|1)$ algebra.

Chiral primaries in the symmetric product CFT satisfy the conditions $G^{ab}_{1/2}|\psi\rangle = \tilde{G}^{ab}_{1/2}|\psi\rangle = G^{+b}_{-1/2}|\psi\rangle = \tilde{G}^{+b}_{-1/2}|\psi\rangle = 0$. They satisfy $L_0 = J^3$ and $\tilde{L}_0 = \tilde{J}^3$ We will focus on chiral primaries which are the ground states in the $\mathbb{Z}_J$ twisted sector with the left and the right J^3 charge given by $(\frac{J-1}{2}, \frac{J-1}{2})$, we denote this chiral primary by $|0\rangle_n \otimes |0\rangle_n$. The construction of this chiral primary ground state using twist operators is given in [14, 15].[2]

We now consider the following excitations above this chiral primary

$$|\phi_{p_1}\phi_{p_2}\cdots\phi_{p_j}\rangle_J \otimes |0\rangle_J = J^-_{p_1} J^-_{p_2} \cdots J^-_{p_j} |0\rangle_J \otimes |0\rangle_J. \tag{2.4}$$

[2]For a detailed review please see [16])

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where J_p^- is given by

$$J_p^- = \sum_{k=1}^{J} e^{ipk} J_{(k)}^-. \qquad (2.5)$$

and $J_{(k)}^-$ is the lowering operator of the left moving SU(2) R-current of the k-th copy of the torus involved in the $\mathbb{Z}_J$ twisted sector. To satisfy orbifold group invariance condition we need to impose the condition

$$\sum_i p_i = 0. \qquad (2.6)$$

At the free orbifold point such a state is non-chiral in the left moving sector while it is still chiral in the right-moving sector. We now perturb the symmetric product CFT with the marginal operator constructed from the $\mathbb{Z}_2$ twist field and which is a singlet component of $SU(2)_I$. This operator is dual to a combination of Ramond-Ramond 0-form and a 4-form on the dual gravity type IIB background. This perturbation is given by [16]

$$\lambda \left(G_{-1/2}^{++} \tilde{G}_{-1/2}^{+-} - G_{-1/2}^{+-} \tilde{G}_{-1/2}^{++} \right) \Sigma^{(1/2,1/2)} + \text{c.c}, \qquad (2.7)$$

where λ is the coupling constant, and $\Sigma^{(1/2,1/2)}$ refers to the $\mathbb{Z}_2$ twist operator of charge $(1/2, 1/2)$. Finally c.c refers to the expression involving the antichiral field $\bar{\Sigma}^{(1/2,1/2)}$. On perturbing the CFT with this operator the excited states given in (2.4) are no longer right-chiral [8][3] it picks up anomalous dimensions.

We now recall the results of the evaluation of the anomalous dimensions of the class of operators given in (2.4) to first order in λ in the limit $J \to \infty$ and $p_i \ll 1$ together with the number of excitations being small [8]. Consider the state[4]

$$|\phi_p\rangle_J \otimes |0\rangle_J = J_p^- |0\rangle_J \otimes |0\rangle_J. \qquad (2.8)$$

1. To first order in λ the action of $\tilde{G}_{+1/2}^{-a}$ flips the state $|\phi_p\rangle_J \otimes |0\rangle_J$ from the $\mathbb{Z}_J$ twisted sector to the $\mathbb{Z}_{J-1}$ twisted sector. We write this as

$$\tilde{G}_{1/2}^{-a} |\phi_p\rangle_J \otimes |0\rangle_J \propto \epsilon^{ab} \lambda G_{-1/2}^{+b} |\phi_p\rangle_{J-1} \otimes |0\rangle_{J-1}. \qquad (2.9)$$

   The above transition clearly conserves the left and right $J^3, \tilde{J}^3$ charge. [8] evaluated the following overlap in the limit $J \to \infty, p \ll 1$ to first order in λ

$$_{J-1}\langle 0| \otimes {}_{J-1}\langle \phi_p | G_{1/2}^{-a} \tilde{G}_{1/2}^{-b} |\phi_p\rangle_J \otimes |0\rangle_J = \epsilon^{ab} \frac{\lambda p \sqrt{Q_1 Q_5}}{4\pi}. \qquad (2.10)$$

   Note that from the equation in (2.9) we see that the state $|\psi_p\rangle_J \otimes |0\rangle_J$ which was chiral on the right movers is no longer chiral.

[3] We follow the reference [8], related work has been done in [6, 7, 17]. The analysis of these build upon the detailed evaluation of the 3-point functions of correlation functions of twist operators for symmetric products which were performed in [18 – 20].

[4] In [8] the momentum of the state in (2.8) is referred to by the label n, with $2\pi n/J = p$.

2. In [8] it was shown that to first order in λ the following commutation relation is obeyed on the state.

$$\left\{\tilde{G}^{-a}_{1/2}, G^{-b}_{1/2}\right\}|\phi_p\rangle_J \otimes |0\rangle_J = \epsilon^{ab}\lambda \int dz\partial_z(z\bar{z}\bar{\Sigma}^{(1/2,1/2)}|\phi_p\rangle_J \otimes |0\rangle_J. \qquad (2.11)$$

Note that the operator $\Sigma^{(1/2,1/2)}$ corresponds to the chiral primary with charge $(1/2,1/2)$, therefore $\int dz\partial_z(z\bar{z}\bar{\Sigma}^{(1/2,1/2)})$ in (2.11) commutes with the following set of generators

$$\left\{\tilde{G}^{-a}_{1/2}, \tilde{G}^{+a}_{-1/2}, G^{-a}_{1/2}, G^{+a}_{-1/2}\right\}.$$

Thus with respect to the SU(1|1) $\times$ SU(1|1) subalgebra given in (2.3) the operator $\int dz\partial_z(z\bar{z}\bar{\Sigma}^{(1/2,1/2)})$ is central. Furthermore the action of the operator evaluated at the leading order in λ and for $p \ll 1$ is given by [8]

$$\int dz\partial_z(z\bar{z}\bar{\Sigma}^{(1/2,1/2)})|\phi_p\rangle_J \otimes |0\rangle_J \sim p|\phi_p\rangle_{J-1} \otimes |0\rangle_{J-1}. \qquad (2.12)$$

Thus on physical states which satisfy (2.6) the action of the central element vanishes. The commutation relation given in (2.11) can also be seen in the plane wave limit [8]. In the plane wave limit of $AdS_3 \times S^3$ it can be seen that [8] the charges obey the following commutation relations

$$\begin{aligned}\left\{G^{-a}_{1/2}, \tilde{G}^{-b}_{1/2}\right\} &= \epsilon^{ab}\frac{1}{p^+}\sum_p pN_p, \\ \left\{G^{+a}_{-1/2}, \tilde{G}^{+b}_{-1/2}\right\} &= \epsilon^{ab}\frac{1}{p^+}\sum_p pN_p.\end{aligned} \qquad (2.13)$$

where N_p is the oscillator number operator at momentum p on the pp wave and p^+ is the light cone momentum. From these commutation relations also it is seen that on physical states, which satisfy the condition $\sum_p pN_p = 0$, the anti-commutation relations vanish.

3. To the leading order in λ the correction to $\Delta - J$ where $\Delta = L_0 + \tilde{L}_0$ and $J = J^3 + \tilde{J}^3$ is given by [8]

$$\Delta - J = 1 + \frac{1}{2\pi^2}\lambda^2(Q_1Q_5)\left(\frac{p^2}{4}\right). \qquad (2.14)$$

4. Note the that marginal deformation of the conformal field theory given in (2.7) is such that $L_0 = \bar{L}_0$. Furthermore the excited state given in (2.8) is such that $L_0 = \bar{L}_0$, therefore in perturbation theory it is clear that the change in conformal weights of states is such that $\delta L_0 = \delta\bar{L}_0$

In principle there could be the following transition from the $\mathbb{Z}_J$ twisted sector to the $\mathbb{Z}_{J+1}$ sector

$$\tilde{G}^{+a}_{-1/2}|\phi_p\rangle_J \otimes |0\rangle_J \to \epsilon^{ab}G^{-b}_{1/2}|\phi_p\rangle_{J+1} \otimes |0\rangle_{J+1}. \qquad (2.15)$$

Note that the $J^3, \tilde{J}^3$ charges are conserved under such transitions, but to first order in λ such transitions are not present [8],

3. The SU(1|1) × SU(1|1) dynamic spin chain model

As we have seen in the previous section that it is possible to obtain the conformal dimensions of the magnon excitations within perturbation theory it is sufficient to restrict our attention to the action of the supercharges $\left\{G^{+-}_{-1/2}, G^{-+}_{1/2}, \tilde{G}^{+-}_{-1/2}\tilde{G}^{-+}_{1/2}\right\}$ or the set of supercharges $\left\{G^{++}_{-1/2}, G^{--}_{1/2}, \tilde{G}^{++}_{-1/2}\tilde{G}^{--}_{1/2}\right\}$. From the commutation relations in (2.2) it can be seen that the above charges generate the subgroup SU(1|1) × SU(1|1) with central charges $L_0 - J^3$ and $\tilde{L}_0 - \tilde{J}^3$. To simplify the discussion we choose one of the SU(1|1) × SU(1|1) algebra and define the generators as follows

$$\begin{aligned} G^{+-}_{-1/2} &\to Q_1, & \tilde{G}^{+-}_{-1/2} &\to Q_2, \\ G^{-+}_{1/2} &\to S_1, & \tilde{G}^{-+}_{1/2} &\to S_2, \\ L_0 - J^3 &\to C_1, & \tilde{L}_0 - \tilde{J}^3 &\to C_2. \end{aligned} \tag{3.1}$$

In terms of these variables, the SU(1|1) × SU(1|1) algebra is given by

$$\begin{aligned} \{Q_1, S_1\} &= C_1, & \{Q_2, S_2\} &= C_2 \\ \{Q_1, Q_2\} &= 0, & \{S_1, S_2\} &= 0, \\ \{Q_1, S_2\} &= 0, & \{S_1, Q_2\} &= 0. \end{aligned} \tag{3.2}$$

C_1 and C_2 are central elements of the algebra.

The magnon excitations given in (2.4) with momentum $p_i = 0$ belong to the BPS states of this algebra with $C_1 = j, C_2 = 0$. We now consider magnons with momentum $p_i \neq 0$, these states are not BPS in the above algebra as $C_1, C_2 \neq 0$. But, on turning on interactions due to the marginal operator in (2.7) we propose that the above algebra gets central extended with 2 more additional central charges. The magnons are then BPS states within this extended algebra and carry these central charges. These central charges are such that on physical states they vanish. We then derive the dispersion relation relating the conformal dimensions of the magnons to the momentum p_i.

3.1 The extended SU(1|1) × SU(1|1) algebra

From the anti-commutation relation (2.11) derived at first order in perturbation theory and the anti-commutation relations (2.13) obtained in the plane wave limit we see that the we should extend the SU(1|1) × SU(1|1) subalgebra such that $\{Q_1, Q_2\}$ and $\{S_1, S_2\}$ is non-trivial. Therefore we consider the following central extension of the SU(1|1) × SU(1|1) algebra, given by the commutation relations

$$\begin{aligned} \{Q_1, S_1\} &= C_1, & \{Q_2, S_2\} &= C_2, \\ \{Q_1, Q_2\} &= C_3 - iC_4, & \{S_1, S_2\} &= C_3 + iC_4, \\ \{Q_1, S_2\} &= 0, & \{S_1, Q_2\} &= 0. \end{aligned} \tag{3.3}$$

Note that we have extended the algebra by including 2 more central charges C_3, C_2, further more since

$$Q_a^\dagger = S_a, \tag{3.4}$$

we have $\{Q_1, Q_2\}^\dagger = \{S_1, S_2\}$ the central charges in these two cases are related by a Hermitian conjugation. Note that the above central extension of $\mathrm{SU}(1|1)\times\mathrm{SU}(1|1)$ is different from that considered in [21] which arises in certain sub-sectors of $\mathcal{N}=4$ Yang-Mills.[5]

This central extension of the $\mathrm{SU}(1|1)\times\mathrm{SU}(1|1)$ in (3.3) can be viewed as a $N=2$ Poincaré superalgebra in 3-dimensions with one central charge. The remaining central charges play the role of 3-momentum in the Poincaré superalgebra. This is similar to the case of giant magnons of $\mathcal{N}=4$ Yang-Mills, there the magnons are BPS states of the central extended SU(2|2) superalgebra which can also be written as a Poincaré superalgebra in 3-dimensions [10]. To view the centrally extended algebra in (3.3) as a $\mathcal{N}=2$ Poincaré algebra we first define the following two component Majorana spinors in 3-dimension

$$q^1 = \begin{bmatrix} Q_1 + S_1 \\ i(Q_2 - S_2) \end{bmatrix}, \qquad q^2 = \begin{bmatrix} i(Q_1 - S_1) \\ (Q_2 + S_2) \end{bmatrix}. \tag{3.5}$$

It is easy to see that from the property (3.4) that these charges are real. We now can write the commutation relations for the extended algebra in (3.3) as

$$\left\{q^i_\alpha, q^j_\beta\right\} = 2\delta^{ij}\hat{p}_\mu(\tilde{\gamma}^\mu)_{\alpha\beta} + 2\epsilon^{ij}\epsilon_{\alpha\beta}C_3. \tag{3.6}$$

Our conventions for the 3-dimensional γ-matrices are as follows:

$$\gamma^0 = i\sigma^2, \qquad \gamma^1 = \sigma^1, \qquad \gamma^2 = \sigma^3. \tag{3.7}$$

where σ^i are Pauli matrices. We also define

$$\tilde{\gamma}^\mu_{\alpha\beta} = (\gamma^\mu)_\alpha^\gamma\epsilon_{\gamma\beta}, \qquad \tilde{\gamma}^0 = -\delta^{\alpha\beta}, \qquad \tilde{\gamma}^1 = -\sigma^3, \qquad \tilde{\gamma}^2 = \sigma^1. \tag{3.8}$$

From (3.3) and the definition of $\tilde{\gamma}^\mu$ and the relation (3.6) we see that the momenta $\hat{p}_\mu$ are identified with the central charges as follows

$$-\hat{p}_0 - \hat{p}_1 = C_1, \qquad -\hat{p}_0 + \hat{p}_1 = C_2, \qquad \hat{p}_2 = C_4. \tag{3.9}$$

The algebra given in (3.6) is the super Poincare algebra in $3d$ with the central charge C_3.The remaining central charges of the extended $\mathrm{SU}(1|1)\times\mathrm{SU}(1|1)$ algebra are identified with the momenta in $3d$ which commute with the supercharges. From the r.h.s. of (3.6) we see that BPS states exist when

$$\hat{p}_0^2 = \hat{p}_1^2 + \hat{p}_2^2 + C_3^2, \quad \text{or} \quad \frac{1}{4}(C_1+C_2)^2 = \frac{1}{4}(C_1-C_2)^2 + C_3^2 + C_4^2. \tag{3.10}$$

3.2 Dynamic spin chain representation

We have seen that the magnon excitations given in (2.4) with $p_i = 0$ are states with $C_1 = j$ and $C_2 = 0$, one can also see that they satisfy the BPS condition (3.10). In this section following [22] we write down a representation of the extended $\mathrm{SU}(1|1)\times\mathrm{SU}(1|1)$ algebra in terms of a dynamic spin chain which carries the central charges C_3, C_4. We

[5] In the extension considered by [21], the anti-commutators $\{Q_1, S_2\}$ and $\{S_1, Q_2\}$ were non-trivial.

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propose that these states correspond to magnons with $p_i \neq 0$. The charges are turned on in such a way that on physical states they vanish. We thus satisfy the property (2.11) and (2.13) seen both in perturbation theory at first order as well as in the plane wave limit. Using this representation of magnons we derive a dispersion relation of the energy $C_1 + C_2 = \Delta - J = (L_0 + \tilde{L}_0) - (J^3 + \tilde{J}^3)$ of these magnons. This dispersion relation satisfies the BPS condition (3.10) and thus is valid at all orders in interaction. In the next section we identify the magnons at strong coupling and show that they indeed satisfy the same dispersion relation.

The vacuum state of the spin chain is a chrial primary denoted as

$$|0\rangle_J \otimes |0\rangle_J = |\ldots\psi\psi\ldots\rangle \otimes |\ldots\tilde{\psi}\tilde{\psi}\ldots\rangle. \tag{3.11}$$

This state represents the chiral primary or weight $(\frac{J-1}{2}, \frac{J-1}{2})$ in the $\mathbb{Z}_J$ twisted sector of the symmetric product. We work in the limit $J \to \infty$. It is convenient to think of each $|\psi\rangle$ as state which carries weight $L_0 = 1/2$ and $J^3 = 1/2$. In the language of the twist field it is a $\mathbb{Z}_2$ twist field which implements the permutation between two copies of the torus T^4. Thus in the vacuum state in (3.11), there are $J-1$ $|\psi\rangle$'s each carrying $L_0 = 1/2, J^3 = 1/2$ at the $J-1$ sites for the right moving vacuum and similarly there are $J-1$ $|\tilde{\psi}\rangle$ which carries weight $\tilde{L}_0 = 1/2, \tilde{J}^3 = 1/2$ at $J-1$ sites for the left moving vacuum. All charges Q_1, Q_2, S_1, S_2 annihilate the vacuum (3.11) since it is a chiral primary. From now on we will work in the limit of the infinite $J \to \infty$ chain. We consider the following excitations on this vacuum

$$|\phi_{p_1}\ldots\phi_{p_j}\rangle \otimes |0\rangle = \sum_{n_1,\ll\ldots\ll n_j} e^{ip_1n_1}\ldots e^{ip_jn_j}|\cdots\psi\psi\cdots\phi_1\cdots\phi_2\cdots\phi_j\cdots\psi\psi\cdots\rangle \otimes |0\rangle. \tag{3.12}$$

Note that we have removed the subscript J from the kets since we are working in the strict $J \to \infty$ limit. Here the state $|\phi\rangle$ represents a state with $L_0 = 1/2, J^3 = -1/2$ with It is obtained from the state $|\psi\rangle$ by the

$$|\phi\rangle = J^-|\psi\rangle. \tag{3.13}$$

Thus excitations given in (3.12) can be obtained following action of J^-_p on the vacuum

$$J^-_{p_1}J^-_{p_2}\cdots J^-_{p_j}|0\rangle \otimes |0\rangle, \tag{3.14}$$

where

$$J^-_p = \sum_l e^{ipl}J^-_{(l)}. \tag{3.15}$$

$J^-_{(l)}$ acts on the state $|\psi\rangle$ at site l. Thus the state in (3.12) corresponds to the state defined in (2.4). The central charges at zeroth order in the coupling of the theory of this state is given by $C_1 = j, C_2 = 0$.

To define the action of the charges Q_a, S_a on the general excited state (3.12) we first define their action on the simple state with one ψ excited to ϕ on the extreme left. Let

$$|\phi\rangle \otimes |0\rangle = |\phi\psi\psi\cdots\rangle \otimes |\tilde{\psi}\tilde{\psi}\cdots\rangle, \tag{3.16}$$

then the action of the charges on this state is given by

$$\begin{aligned} Q_1|\phi\rangle \otimes |0\rangle &= a|\psi^+\phi\rangle \otimes |0\rangle, \\ Q_2|\phi\rangle \otimes |0\rangle &= a'|\phi\rangle \otimes |\tilde{\psi}^+\rangle, \\ S_1|\phi\rangle \otimes |0\rangle &= b|\psi^-\phi\rangle \otimes |0\rangle, \\ S_2|\phi\rangle \otimes |0\rangle &= b'|\phi\rangle \otimes |\tilde{\psi}^-\rangle. \end{aligned} \tag{3.17}$$

In the above equations the presence of ψ^+ refers to the fact that that state is in the $\mathbb{Z}_{J+1} \times \mathbb{Z}_J$ twisted sector, while the presence of ψ^- refers to the fact that the state is in the $\mathbb{Z}_{J-1} \times \mathbb{Z}_J$ twisted sector. Similarly the presence of $\tilde{\psi}^+$ refers to the fact that that the state is in the $\mathbb{Z}_J \otimes \mathbb{Z}_{J+1}$ twisted sector, and the presence of $\tilde{\psi}^-$ refers to the fact that the state is in the $\mathbb{Z}_J \otimes \mathbb{Z}_{J-1}$ sector. a, a', b, b' are constants which depend on the interaction strength of the theory and should vanish at the zeroth order in coupling. These transition rules are motivated from the observations given in (2.9) and (2.10) seen in the symmetric product conformal field theory at first order in λ. In the extended algebra (3.3) we have $Q_1^2 = 0, S_1^2 = 0, Q_2^2 = 0, S_2^2 = 0$. We write this as

$$\begin{aligned} Q_1|\psi^+\phi\rangle \otimes |0\rangle &= 0, \\ Q_2|\phi\rangle \otimes |\tilde{\psi}^+\rangle &= 0, \\ S_1|\psi^-\phi\rangle \otimes |0\rangle &= 0, \\ S_2|\phi\rangle \otimes |\psi^-\rangle &= 0. \end{aligned} \tag{3.18}$$

From the above rules it is clear that the the difference of the twists in the between the left and the right moving sectors can be at the most ± 1. To impose the anti-commutation relations $\{Q_1, S_2\} = 0, \{Q_2, S_2\} = 0$ on the states of spin chain we assume the following states in the spin chain are proportional.

$$Q_1|\Psi\rangle = gS_2|\Psi\rangle, \qquad Q_2|\Psi\rangle = g'S_1|\Psi\rangle. \tag{3.19}$$

where $|\Psi\rangle$ is any state obtained by the action of charges on the state $|\phi\rangle \otimes |0\rangle$. The above equation is motivated by the observation (2.9) seen in perturbation theory. It is clear that using the above equation and the fact $Q_1^2 = 0, S_1^2 = 0, Q_2^2 = 0, S_2^2 = 0$ the anti-commutation relations $\{Q_1, S_2\} = 0, \{Q_2, S_2\} = 0$ on the excited states are seen to hold. Now the only other non-trivial sequences of action of charges to specify the representation are $Q_1Q_2, Q_2Q_1, S_1S_2, S_2S_1, Q_1S_1, S_1Q_1, Q_2S_2, S_2Q_2$. We write these

$$\begin{aligned} Q_1Q_2|\phi\rangle \otimes |0\rangle &= aa'|\psi^+\rangle \otimes |\tilde{\psi}^+\rangle, \\ Q_2Q_1|\phi\rangle \otimes |0\rangle &= aa'\gamma|\psi^+\rangle \otimes |\tilde{\psi}^+\rangle, \\ S_1S_2|\phi\rangle \otimes |0\rangle &= bb'|\psi^-\phi\rangle \otimes |\tilde{\psi}^-\rangle, \\ S_2S_1|\phi\rangle \otimes |0\rangle &= bb'\gamma'|\psi^-\phi\rangle \otimes |\tilde{\psi}^-\rangle, \\ S_1Q_1|\phi\rangle \otimes |0\rangle &= ab|\phi\rangle \otimes |0\rangle, \\ Q_1S_1|\phi\rangle \otimes |0\rangle &= ab\tilde{\gamma}|\phi\rangle \otimes |0\rangle, \end{aligned} \tag{3.20}$$

$$
\begin{aligned}
S_2Q_2|\phi\rangle \otimes |0\rangle &= a'b'|\phi\rangle \otimes |0\rangle, \\
Q_2S_2|\phi\rangle \otimes |0\rangle &= a'b'\tilde{\gamma}'|\phi\rangle \otimes |0\rangle.
\end{aligned}
$$

Note that the action of the Q_1Q_2 does not anti-commute with the action of Q_2Q_1 for $\gamma \neq -1$ this is precisely what we require if the central charges C_3, C_4 need to be turned on. A similar statement holds for the action of S_1S_2 and S_2S_1. Thus interchanging Q_1Q_2 picks up a factor of γ and interchanging S_1S_2 picks up a factor of γ'. Similarly note that Q_1S_1 and Q_2S_2 also do not anti-commute.

Now using the defintion of the spin chain representation given in (3.17), (3.18) and (3.20) we read out the central charges carried by the representation. Examining the relations $\{Q_1, S_1\}$ and $\{Q_2, S_2\}$ on the excited state in (3.16) give

$$
\begin{aligned}
C_1|\phi\rangle \otimes |0\rangle &= ab(1+\tilde{\gamma})|\phi\rangle \otimes |0\rangle, \\
C_2|\phi\rangle \otimes |0\rangle &= a'b'(1+\tilde{\gamma}')|\phi\rangle \otimes |0\rangle.
\end{aligned} \tag{3.21}
$$

Finally from the relations $\{Q_1, Q_2\}$ and $\{S_1, S_2\}$ we have

$$
\begin{aligned}
(C_3 - iC_4)|\phi\rangle \otimes |0\rangle &= (1+\gamma)aa'|\psi^+\phi\rangle \otimes |\tilde{\psi}^+\rangle, \\
(C_3 + iC_4)|\phi\rangle \otimes |0\rangle &= (1+\gamma')bb'|\psi^-\phi\rangle \otimes |\tilde{\psi}^-\rangle.
\end{aligned} \tag{3.22}
$$

In deriving this we used the rules given in (3.20) Note that from the action of the charges given in (3.17), (3.19), (3.18) and (3.20) it can be shown that the charges C_3, C_4 are central. Therefore the rules (3.17) , (3.19), (3.18) and (3.20) define a representation of the extended SU(1|1) × SU(1|1) algebra.

We are interested in representation such that the central charges C_3, C_4 are turned on locally in a state but globally on physical states these central charges vanish. For this we consider an excitation of definite momentum p given by

$$
|\phi_p\rangle \otimes |0\rangle = \sum_k e^{ipk}|\ldots\psi\psi\ldots\phi\ldots\psi\psi\ldots\rangle \otimes |0\rangle. \tag{3.23}
$$

Insertion or removal of ψ to the immediate left of the excitation we obtain

$$
\begin{aligned}
|\psi^\pm\phi_p\rangle \otimes |0\rangle &= \sum_k e^{ipk}|\ldots\psi^\pm\phi\ldots\rangle \otimes |0\rangle, \\
&= \sum_k e^{ipk\mp ip}|\ldots\psi\psi\ldots\phi\ldots\rangle \otimes |0\rangle.
\end{aligned} \tag{3.24}
$$

Thus upto a phase we can shift the insertion or removal of ψ to the very end. We therefore have the relation

$$
|\psi^\pm\phi_p\rangle \otimes |0\rangle = e^{\mp ip}|\phi\psi^\pm\rangle \otimes |0\rangle. \tag{3.25}
$$

Thus the action of the central charge $C_3 - iC_4$ on the tensor product of excitations is given by

$$
\begin{aligned}
(C_3 - iC_4)|\phi_{p_1}\ldots\phi_{p_j}\rangle \otimes |0\rangle &= \mathcal{C}|\phi_{p_1}\ldots\phi_{p_j}\psi^+\rangle \otimes |\psi^+\rangle, \\
\mathcal{C} &= \sum_{k=1}^{j} a_k a'_k(1+\gamma) \prod_{l=k+1}^{j} e^{-ip_l}.
\end{aligned} \tag{3.26}
$$

Therefore $\mathcal{C}$ should vanish on physical states, this is obtained by setting

$$a_k a'_k(1+\gamma) = \alpha\left(e^{-ip_k} - 1\right), \tag{3.27}$$

where α is a function of the coupling $\tilde{\lambda}$, the normalization of α above is for convenience. With this condition it is easy to see that $\mathcal{C}$ is given by

$$\mathcal{C} = \alpha\sum_{k=1}^{j}(e^{-ip_k} - 1)\prod_{l=k+1}^{j} e^{-ip_l} = \alpha\left(\prod_{k=1}^{j} e^{-ip_k} - 1\right). \tag{3.28}$$

Thus on physical state $\mathcal{C}$ vanishes. Similarly we see that we should set

$$b_k b'_k(1+\gamma') = \alpha^*\left(e^{ip_k} - 1\right), \tag{3.29}$$

which ensures $C_3 + iC_4$ also vanishes on physical state. Note that the central charge $C_3 + iC_4$ is the complex conjugate of $C_3 - iC_4$ which has been implemented in (3.27) (3.29). For the state $|\phi_p\rangle \otimes |0\rangle$ we have $C_1 - C_2 = 1$, since

$$C_1 - C_2 = (L_0 - J^3) - (\tilde{L} - \tilde{J}^3) = \tilde{J}^3 - J^3. \tag{3.30}$$

Here the last equality follows due to property 4. seen in perturbation theory, that is the condition $L_0 = \tilde{L}_0$ is maintained in perturbation theory. from (3.21) we have

$$(1+\tilde{\gamma})ab - (1+\tilde{\gamma}')a'b' = 1. \tag{3.31}$$

Then from the equations (3.27), (3.29) and (3.31) we obtain

$$\begin{aligned} \Delta - J = C_1 + C_2 &= \left((1+\tilde{\gamma})ab + (1+\tilde{\gamma}')a'b'\right) \\ &= \sqrt{1 + 16\alpha^*\alpha\frac{(1+\tilde{\gamma})(1+\tilde{\gamma}')}{(1+\gamma)(1+\gamma')}\sin^2\left(\frac{p}{2}\right)}, \\ &= \sqrt{1 + f(\tilde{\lambda})\sin^2\left(\frac{p}{2}\right)}. \end{aligned} \tag{3.32}$$

We thus have obtained the dispersion relations for the magnons. Note that the above relation satisfies the BPS condition given in (3.10). From the comparison of the correction to $\Delta - J$ computed at weak coupling and small mometum given in (2.14) we see that

$$f(\tilde{\lambda}) = \lambda^2\frac{Q_1 Q_5}{\pi^2}. \tag{3.33}$$

For the tensor product exicitation given in (3.12) with the assumption that the exciations are well separated we obtain the dispersion relation

$$\Delta - J = \sum_{i=1}^{j}\sqrt{1 + f(\tilde{\lambda})\sin^2\left(\frac{p_i}{2}\right)}. \tag{3.34}$$

4. Magnons at strong coupling

The classical solutions of the string sigma model found by [10] for the case of $AdS_5 \times S^5$ continue to be solutions in $AdS_3 \times S^3 \times T^4$. This is because they require only the subspace $R \times S^2$ which is also available in $AdS^3 \times S^3$. It is only when there is Ramond-Ramond flux through the S^3 the equations of motion discussed by [10] continue to be the same for the case of $AdS^3 \times S^3$. Thus the discussion in this section applies for pure Ramond-Ramond flux through the S^3. We first start with a short review of the magnon solution and obtain the dispersion relation of the magnons at strong coupling. We then provide a detailed analysis of the supersymmetry preserved by these magnons by following the logic outlined in [10]. This involves writing the solution in LLM like coordinates for the case of $AdS_3 \times S^3$ in which the magnon solution is just a stretched string. By studying the Killing spinors and a particular one-form which corresponds to the gauge transformation of the NS B-form under the action of two supersymmetries we see that the magnon solution carries the required central charges to be BPS. The reason it is BPS is the same as the reason a stretched string is BPS in flat space, in fact the supersymmetry algebra turns out to be the the extended SU(1|1) × SU(1|1) algebra discussed in in section 2.

4.1 Magnon dispersion relation at strong coupling

The near horizon geometry of the D1-D5 system for large $\tilde{\lambda}$ is $AdS_3 \times S^3 \times T^4$ described by the following metric [3]:

$$ds^2 = R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\varphi^2 + d\Omega_3^2) + ds^2([T^4]), \tag{4.1}$$

where $d\Omega_3^2$ is the metric on the unit three sphere given by

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2, \tag{4.2}$$

with $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, $0 \le \psi \le 2\pi$, and $ds^2([T^4]$ is the flat metric on the four torus given by

$$ds^2[T^4] = \alpha' \sqrt{\frac{Q_1}{vQ_5}} \left(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2\right). \tag{4.3}$$

v is the volume of asymptotic volume of the four torus in string units, and

$$R^2 = \alpha' g_6 \sqrt{Q_1 Q_5}, \tag{4.4}$$

with $g_6 = g_s/\sqrt{v}$. Note that we have used global coordinates to describe AdS_3. We identify ϕ to be the coordinate conjugate to angular momentum $J = J^3 + \tilde{J}^3$. The string ground state with $E - J = 0$ corresponds to a lightlike trajectory that moves along ϕ with $\phi - t =$ constant and sits at $\theta = \pi/2$. and at the center of AdS_3, $\rho = 0$. Now to obtain the string configuration that corresponds to a solution carrying momentum p with the least amount of energy $\epsilon = E - J$ we follow [10]. We first choose the point $\psi = 0$ on the circle S^1 parameterized by ψ. This point along with θ and ϕ form a S^2. After we include time, the motion takes place in $R \times S^2$ where R is parameterized by the time coordinate. We now write the Nambu-Goto action for the string in this background by choosing the world sheet coordinates to be:

$$t=\tau, \qquad \phi - t = \sigma, \tag{4.5}$$

and we consider a configuration where θ is independent of τ. The Nambu-Goto action takes the form:

$$S=\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-det\gamma}, \tag{4.6}$$

where γ is the induced metric on the world sheet given by

$$\gamma_{ab}=g_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}, \tag{4.7}$$

where $a, b = 0, 1$ and $\sigma^0 = \tau$ and $\sigma^1 = \sigma$. After taking into account the worldsheet parameterization given in (4.5) we get the following action,

$$S=\frac{R^2}{2\pi\alpha'}\int d\tau d\sigma\sqrt{\cos^2\theta\theta'^2+\sin^2\theta}, \tag{4.8}$$

here θ' refers to derivative with respect to σ. Following [10] we integrate the equations of motion and get

$$\sin\theta=\frac{\sin\theta_0}{\cos\sigma}, \qquad -\left(\frac{\pi}{2}-\theta_0\right)\leq\sigma\leq\frac{\pi}{2}-\theta_0, \tag{4.9}$$

where $0 \leq \theta_0 \leq \frac{\pi}{2}$ is an integration constant. The difference in angle between the two endpoints of the string at a given time $\delta\sigma = 2\left(\frac{\pi}{2}-\theta_0\right)$ is identified with the momentum p of the magnon [10], we write this as

$$\delta\phi=2\left(\frac{\pi}{2}-\theta_0\right)=p, \tag{4.10}$$

and one also obtains the energy $E - J$ which is the Noether charge corresponding to shifts in σ to obtain

$$E-J=\frac{R^2}{\pi\alpha'}\cos\theta_0=\frac{R^2}{\pi\alpha'}\sin\frac{\delta\phi}{2}. \tag{4.11}$$

After we have identified $\delta\phi$ with p, we obtain the following strong coupling result of the dispersion relation,

$$E-J=\frac{R^2}{\pi\alpha'}\left|\sin\frac{p}{2}\right|, \tag{4.12}$$

Note that this dispersion relation agrees with the strong coupling limit of (1.5) if $f(\tilde{\lambda}) \to g_6^2 Q_1 Q_5/\pi^2$ for $\tilde{\lambda} \to \infty$. We now proceed to demonstrate that these magnons are supersymmetric.

4.2 Supersymmetry preserved by magnons

There are two crucial ingredients to demonstrate that these magnons are BPS solutions of type IIB on $AdS_3 \times S^3 \times T^3$. The first one is to demonstrate that in a particular coordinate system the magnon solution is just a straight stretched string. For this we write the solution given in (4.9) using the LLM [12] coordinates suitable for $AdS_3 \times S^3$. The $AdS_3 \times S^3$ metric in these coordinates is given by [13]

$$ds_6^2=-h^2(dt+V_idx^i)^2+h^2(dy^2+\delta_{ij}dx^idy^j)+y(e^Gd\Omega_1^2+e^{-G}d\tilde{\Omega}_1^2), \tag{4.13}$$

where

$$h^{-2} = 2y \cosh G, \qquad z \equiv \frac{1}{2} \tanh G, \qquad dV = -\frac{1}{y} *_3 dz. \tag{4.14}$$

and z satisfies the following equations

$$\begin{aligned} \left(\partial_i^2 + y \partial_y \frac{1}{y} \partial_y \right) z &= 0, \\ \partial_i z \partial_i z + \partial_y z \partial_y z &= \frac{(1 - 4z^2)^2}{4y^2}. \end{aligned} \tag{4.15}$$

For the $AdS_3 \times S^3$ metric z is such that in the plane $y = 0$, $z = 1/2$ is a circular region of radius R. The above metric is a fibration of the time direction t and the two S^1's denoted by Ω_1 and $\tilde{\Omega}_1$ over the three dimensional space characterized by x_1, x_2, y. We can obtain the conventional global coordinates of $AdS_3 \times S^3$ given in (4.1) using the following change of coordinates

$$\begin{aligned} y &= \sinh \rho \cos \theta, & r &= \cosh \rho \sin \theta, & \sigma &= \phi - t, \\ x_1 &= r \cos \sigma, & x_2 &= r \sin \sigma, & & \\ \varphi &= \Omega_1, & \psi &= \tilde{\Omega}_1. & & \end{aligned} \tag{4.16}$$

Using this change of variables, the metric on the plane $y = 0$ for $r < 1$ is of the form

$$ds^2 = R^2 \left[-(1 - r^2) \left(dt - \frac{r^2}{1 - r^2} d\sigma \right)^2 + \frac{dr^2 + r^2 d\sigma^2}{1 - r^2} + (1 - r^2) d\psi^2 \right]. \tag{4.17}$$

We now repeat the analysis of [10] for this case. From (4.16) we see that $r^2 = \sin^2 \theta = x_1^2 + x_2^2$, since $\rho = 0$, the solution (4.9) can be written as

$$r \cos \sigma = x_1 = \cos \theta_0 = \text{constant}. \tag{4.18}$$

Thus the magnon solution is just a straight stretched string in these coordinates. The string is stretched between two points on a circle. Note that the energy $E - J$ of the magnon is just the length of the string with flat metric on (x_1, x_2) plane,

$$E - J = \frac{R^2}{\alpha' \pi} \Delta x_2 = \frac{R^2}{\alpha' \pi} \cos \theta_0. \tag{4.19}$$

Finally in these coordinates the angle subtended by the string at the centre of the circle is related to p by $2\theta_0 = \pi - p$ from (4.10)

The second ingredient needed to show that the giant magnon solution preserves supersymmetry is to demonstrate the it carries the appropriate central charges. These central charges arise due to the fact that it is just a straight stretched string in the LLM like coordinates and thus it has the appropriate winding charge needed to make it supersymmetric. This is the same reason that stretched strings in flat space are BPS. To show that that the giant magnon solution carries these charges we again follow the logic outlined in [10]. We first need to find out the Killing spinors for the $AdS_3 \times S^3$ solution in LLM like coordinates.

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We start with the LLM like ansatz for type IIB with $S^1 \times S^1$ isometry, this ansatz accommodates $AdS_3 \times S^3 \times T^4$ as a solution [13]. In the near horizon geometry of the D1-D5 system the Ramond-Ramond 3-form is self dual, therefore we take $D = 10$, IIB supergravity with the following bosonic fields turned on: the metric G_{MN}, the 2-form potential C^+_{MN} with self dual field strength i.e $F_{(3)} = *F_{(3)}$ where $F_3 = dC^+_{(2)}$. Let ψ_M be the gravitino which is a right handed Weyl spinor i.e it obeys the condition $\Gamma_{11}\psi_M = \psi_M$, λ is the dilatino which is also a right handed Weyl spinor. Notice that we have set the Ramond-Ramond 4-form potential and axion-dilaton to zero. We specify a reduction of the form:

$$
\begin{aligned}
ds^2_{10} &= g_{\mu\nu}dx^\mu dx^\nu + e^{H(x)+G(x)}d\phi^2 + e^{H(x)-G(x)}d\tilde{\phi}^2 + dx_s dx^s, \\
F_{(3)} &= -\frac{1}{2}F_{(2)} \wedge d\phi - \frac{1}{2}\tilde{F}_{(2)} \wedge d\tilde{\phi},
\end{aligned} \tag{4.20}
$$

where

$$
F_{(2)} = F_{\mu\nu}dx^\mu \wedge dx^\nu, \tag{4.21}
$$

$s = 6, 7, 8, 9$ the directions along T^4 and $\mu = 0, 1, 2, 3$. The above ansatz preserves the $\mathrm{SO}(2) \times \mathrm{SO}(2)$ isometry. This ansatz corresponds to setting the gauge fields from the components of the metric and the 2-form potential : $g_{\mu\phi}$ and $C_{\mu\tilde{\phi}}$ and the scalars: $C^+_{\phi\tilde{\phi}}$ and $g_{\phi\tilde{\phi}}$ components) to zero. This is an inconsistent truncation of the theory but as argued in [13], this inconsistency manifests itself in one additional constraint given in the second line of (4.15). Since we have set the axion-dilaton to a constant and the Ramond-Ramond 4-form potential to zero, the supersymmetry variation of dilatino and gravitino takes the following form [13, 23].

$$
\begin{aligned}
\delta\psi_M &= \nabla_M\varepsilon - \frac{1}{96}(\Gamma_M\Gamma^{NPQ} + 2\Gamma^{NPQ}\Gamma_M)F_{NPQ}\varepsilon^*, \\
\delta\lambda &= -\frac{i}{24}F_{MNP}\Gamma^{MNP}\varepsilon.
\end{aligned} \tag{4.22}
$$

We use the convention that 10d gamma matrices are purely imaginary, explictly they are given in (A.1). Setting the dilatino variation to zero gives the 6d chirality condition on the spinor

$$
\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\epsilon = -\epsilon. \tag{4.23}
$$

Since the spinor ϵ is a 10 Weyl spinor we also have the following condition

$$
\Gamma^6\Gamma^7\Gamma^8\Gamma^9\epsilon = -\epsilon. \tag{4.24}
$$

The gravitino variation in the $0, 1, 2, 3, \phi, \tilde{\phi}$ directions are given by

$$
\begin{aligned}
\delta\psi_\mu &= \nabla_\mu\epsilon - \frac{1}{16}\gamma_{\rho\sigma}F^{\rho\sigma}e^{-\frac{1}{2}(H-G)}\gamma_\mu\tilde{\varepsilon}\hat{\sigma}_1\epsilon^*, \\
\delta\Omega_H &= \frac{1}{2}(\partial_\mu H\gamma^\mu\tilde{\varepsilon}\hat{\sigma}_1)\epsilon + e^{\frac{H+G}{2}}\partial_\phi\epsilon - ie^{-\frac{H-G}{2}}\gamma^5\partial_{\tilde{\phi}}\epsilon, \\
\delta\Omega_G &= \frac{1}{2}(\partial_\mu G\gamma^\mu\tilde{\varepsilon}\hat{\sigma}_1)\epsilon + e^{-\frac{H+G}{2}}\partial_\phi\epsilon + ie^{-\frac{H-G}{2}}\gamma^5\partial_{\tilde{\phi}}\epsilon - \frac{1}{8}\gamma_{\rho\sigma}F^{\rho\sigma}\epsilon^*.
\end{aligned} \tag{4.25}
$$

Here $\delta\Omega_H$ and $\delta\Omega_G$ are defined as

$$
\delta\Omega_H = \delta\psi_\phi - e^G\Gamma_5\Gamma_4\delta\psi_{\tilde{\phi}}, \qquad \delta\Omega_G = \delta\psi_\phi + e^G\Gamma_5\Gamma_4\delta\psi_{\tilde{\phi}}. \tag{4.26}
$$

Note that these gravitino variations are different from that obtained by [13] for the case of 6d $(1,0)$ gravity. Due to the the occurrence of ϵ^* on the r.h.s. of the variations the solutions of the Killing spinors obtained by [13] for the case of 6d $(1,0)$ gravity cannot be directly embedded in IIB gravity. But as shown in appendix A. for the following spinor

$$\tilde{\epsilon} = \epsilon_R + i(1 \otimes \tilde{\varepsilon} \otimes \hat{\sigma}_1)\epsilon_R. \tag{4.27}$$

the gravitino variations reduce to that obtained by [13]. In (4.27) subscripts stand for the real part of the spinor ϵ. Rewriting the gravitino variation (4.25) in terms of the spinor $\tilde{\epsilon}$ from, we get the following equations

$$\begin{aligned}
\delta\tilde{\psi}_\mu &= \nabla_\mu\tilde{\epsilon} - \frac{i}{16}e^{-\frac{1}{2}(H-G)}\gamma_{\rho\sigma}F^{\rho\sigma}\gamma_\mu\tilde{\epsilon} \\
\delta\tilde{\Omega}_H &= -i\frac{1}{2}\partial_\mu H\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\partial_\phi\tilde{\epsilon} - ie^{-\frac{H-G}{2}}\gamma^5\partial_{\tilde{\phi}}\tilde{\epsilon} \\
\delta\tilde{\Omega}_G &= -i\frac{1}{2}\partial_\mu G\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\partial_\phi\tilde{\epsilon} + ie^{-\frac{H+G}{2}}\gamma^5\partial_{\tilde{\phi}}\tilde{\epsilon} - \frac{1}{8}\gamma_{\rho\sigma}F^{\rho\sigma}\tilde{\epsilon}.
\end{aligned} \tag{4.28}$$

We now can take the Kaluza-Klein ansatz for the spinor $\tilde{\epsilon}$

$$\tilde{\epsilon}(x,\phi,\tilde{\phi}) = \exp\left(-\frac{i}{2}(\eta\phi + \tilde{\eta}\tilde{\phi})\right)\tilde{\epsilon}(x). \tag{4.29}$$

Substituting this ansatz in the equations (4.28) we obtain

$$\begin{aligned}
\delta\tilde{\psi}_\mu &= \nabla_\mu\tilde{\epsilon} - \frac{i}{16}e^{-\frac{1}{2}(H-G)}\gamma_{\rho\sigma}F^{\rho\sigma}\gamma_\mu\tilde{\epsilon} \\
i\delta\tilde{\Omega}_H &= \partial_\mu H\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\eta\tilde{\epsilon} - ie^{-\frac{H-G}{2}}\gamma^5\tilde{\eta}\tilde{\epsilon} \\
i\delta\tilde{\Omega}_G &= \partial_\mu G\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\eta\tilde{\epsilon} + ie^{-\frac{H+G}{2}}\gamma^5\tilde{\eta}\tilde{\epsilon} - \frac{i}{4}\gamma_{\rho\sigma}F^{\rho\sigma}\tilde{\epsilon}.
\end{aligned} \tag{4.30}$$

These equations are now identical to the gravitino variation obtained by [13] for the case of 6d $(1,0)$ gravity. As a by product we have obtained the procedure to embed all the solutions obtained by [13] for 6d $(1,0)$ gravity in type IIB gravity. In appendix A.1. we have explicitly written down two Killing spinors in $AdS_3 \times S^4 \times T^4$ in LLM coordinates referred to by

$$\epsilon_1 = \frac{1}{2}(\tilde{\epsilon} + \tilde{\epsilon}^*), \qquad \epsilon_2 = \frac{i}{2}(\tilde{\epsilon}' + \tilde{\epsilon}'^*), \tag{4.31}$$

where $\tilde{\epsilon}$ and $\tilde{\epsilon}'$ are given in (A.44) and (A.45) respectively.

From the study of the supersymmetry algebra of type IIB gravity in [23] it is seen that the action of anti-commutator of two supersymmetries contains a gauge transformation on the Neveu-Schwarz B-field. This implies that under the presence of stretched strings the anti-commutator of the supercharges contains a term proportional to the winding charge of the stretched strings. We write this as

$$Q = \int d^9x\sqrt{-g}j^{M0}\omega_M, \tag{4.32}$$

where the winding current j^{MN} is given by

$$j^{MN}(x) = \frac{R^2}{2\pi\alpha'}\int_M d\tau d\sigma(\partial_\tau X^M\partial_\sigma X^N - \partial_\tau X^N\partial_\sigma X^M)\frac{\delta^{10}(x - X(\sigma))}{\sqrt{-g}}. \tag{4.33}$$

and ω_M is the resulting gauge transformation parameter from the anti-commutator of two supersymmetries. Note that the winding charge (4.32) is conserved only if ω_M is a closed 1-form since $\nabla_M(j^{MN}\omega_M) = \nabla_M j^{MN}\omega_N + j^{MN}\nabla_M\omega_N = 0$ only if $\nabla_M\omega_N - \nabla_N\omega_M$ vanishes. Thus what remains to be done is to determine the relevant gauge transformation parameter which results from the action of the anti-commutator of two supersymmetries and show that that it is a closed form. From the analysis done in appendix A. and appendix B. the relevant gauge transformation parameter is given by $\omega_\mu = i(\bar\epsilon_1\gamma_\mu\epsilon_2)$ where ϵ_1 and ϵ_2 are given in (4.31). In appendix B. we have explicitly evaluated ω_μ and we see that it is a constant closed 1-from. The only non zero components of of ω_μ is given by

$$\omega_1 = \cos\chi, \qquad \omega_2 = \sin\chi, \tag{4.34}$$

where χ is the angle corresponding to the rotation degree of freedom in the $(1,2)$ plane. Since there is a freedom of choosing the angle χ the winding charge of a straight string in the x_1, x_2 plane along any direction is conserved and appears on the r.h.s. of the supersymmetric algebra. All one has to do is to choose χ so that the 1-form (ω_1, ω_2) is along the direction of the string. The magnitude of the winding charge along with its direction in the $(1,2)$ plane has 2 independent components, the straight stretched string carries two additional central charges. For the giant magnon solution given in (4.18) the winding charge using the definition in (4.32), (4.33) is given by

$$\hat{Q} = \frac{R^2}{\pi\alpha'}\cos\theta_0\hat{x}_2, \tag{4.35}$$

the $\hat{x}_2$ denotes the direction of the winding. Thus subalgebra relevant for the giant magnons, $\mathrm{SU}(1|1)\times\mathrm{SU}(1|1)$ is therefore extended by two additional central charges. From the discussion of the extended algebra in section 3, we see that the extended charges also form a vector given by $C_3 + iC_4$. We now identify the central charges as

$$\frac{\hat{Q}}{2} = (C_3 + iC_4). \tag{4.36}$$

The proportionality constant is fixed by the fact that the dispersion relation obtained from the BPS condition (3.10) is consistent with the strong coupling dispersion relation obtained in (4.12). Since the magnons are straight stretched strings and carry the appropriate winding charges they are BPS. Therefore we can apply the BPS relation (3.10) to derive the dispersion relation. For a single magnon this gives

$$\begin{aligned}\Delta - J = (C_1 + C_2) &= \sqrt{1 + \left(\frac{R^2}{\alpha'\pi}\right)^2\cos^2\theta_0}, \\ &= \sqrt{1 + \left(\frac{R^2}{\alpha'\pi}\right)^2\sin^2\frac{p}{2}}.\end{aligned} \tag{4.37}$$

In the second line we have used the identification given in (4.10) of θ_0 with the magnon momentum. Substituting the value of R^2 in terms of the D1, D5-brane charges we obtain

$$\Delta - J = \sqrt{1 + \left(\frac{g_6^2 Q_1 Q_5}{\pi^2}\right)\sin^2\frac{p}{2}}. \tag{4.38}$$

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Thus we see that at strong coupling

$$f(\tilde{\lambda}) = \frac{g_6^2 Q_1 Q_5}{\pi^2}, \qquad \tilde{\lambda} \gg 1. \tag{4.39}$$

5. Discussion

In this paper we have used the centrally extended $\mathrm{SU}(1|1) \times \mathrm{SU}(1|1)$ superalgebra with two more additional central charges to derive the dispersion relation of magnons in the D1-D5 system. The derivation closely followed the derivation of the dispersion relation of magnons in $\mathcal{N} = 4$ Yang-Mills. This similarity suggests that just as $\mathcal{N} = 4$ Yang-Mills is integrable at the planar limit, the D1-D5 system might be integrable for $Q_1 Q_5 \gg 1$. In fact the classical string on $AdS_3 \times S^3$ with Ramond-Ramond flux through the S^3 is known to have infinite set of non-local, commuting conserved charges [24]. Thus using integrability along with the extended symmetry we have found in this paper and proceeding algebraically along the lines of [22] it might be possible to obtain the S-matrix of this theory which will lead to the information about the complete spectrum in the large J limit. Another approach is to look at the S-matrix of magnons in the strong coupling limit. As we have seen at strong coupling the giant magnon solution in the D1-D5 system is identical to that of $\mathcal{N} = 4$ Yang-Mills. Since the evaluation of the S-matrix for scattering of two magnons with momentum p_1 and p_2 at strong coupling just depends on the classical solution, the S-matrix evaluated by [10] applies for magnons in $AdS_3 \times S^3$ as well. This is given by

$$\begin{aligned} &S(p_1, p_2) = \exp(i\delta), \\ &\text{where } \; \delta = -\frac{\sqrt{\lambda}}{\pi}\left(\cos\frac{p_1}{2} - \cos\frac{p_2}{2}\right) \log\left(\frac{\sin^2\frac{p_1 - p_2}{2}}{\sin^2\frac{p_1+p_2}{2}}\right) \end{aligned} \tag{5.1}$$

where $\mathrm{sign}(\sin\frac{p_1}{2}) > 0$ and $\mathrm{sign}(\sin\frac{p_1}{2}) > 0$ and $\lambda = g_6^2 Q_1 Q_5$. It will be interesting to perform the sub-leading corrections to this S-matrix for the $AdS_3 \times S^3$ case following [25], since these depend on small fluctuations around the giant magnon background. Here the fact that we are in the $AdS_3 \times S^3 \times T^4$ background will play a role. The sub-leading corrections to this S-matrix and the use of the extended symmetries we have found in this work might help to determine the complete S-matrix.

Finally, we have studied the extended supersymmetry of the giant magnons following the approach of [10]. It will be interesting to study this issue using the more direct world sheet approach of [26].

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A. Embedding $(1,0)$ 6d supergravity in IIB supergravity

In this section we embed the solutions of $(1,0)$ 6d supergravity found in [13] in type IIB supergravity. The strategy we follow is to rewrite the type IIB supersymmetry variations in terms of supersymmetry variations of $(1,0)$ 6d supergravity given in [13]. This allows us to easily solve for the Killing spinors for $AdS_3 \times S^3$ in the LLM coordinates. We first choose the following convention of 10d gamma matrices.

$$
\begin{aligned}
\Gamma^\mu &= -i\gamma^\mu \otimes \varepsilon \otimes 1 \otimes \hat{\sigma_3}, \\
\Gamma^4 &= i1 \otimes \varepsilon \otimes \tilde{\varepsilon} \otimes \hat{\varepsilon}, \\
\Gamma^5 &= i1 \otimes 1 \otimes \tilde{\sigma_1} \otimes \hat{\varepsilon}, \\
\Gamma^6 &= i1 \otimes 1 \otimes \tilde{\sigma_3} \otimes \hat{\varepsilon}, \\
\Gamma^7 &= i1 \otimes \sigma_1 \otimes \tilde{\varepsilon} \otimes 1, \\
\Gamma^8 &= i1 \otimes \sigma_3 \otimes \tilde{\varepsilon} \otimes 1, \\
\Gamma^9 &= i1 \otimes \varepsilon \otimes 1 \otimes \hat{\sigma_1}.
\end{aligned} \tag{A.1}
$$

Here γ^μ with $\mu = 0,1,2,3$ are 4×4 gamma matrices in the Majorana representation, we write them down explictly below.

$$
\begin{aligned}
\gamma^0 &= -i\sigma_2 \otimes 1, & \gamma^1 &= \sigma^1 \otimes 1, \\
\gamma^2 &= \sigma^3 \otimes \sigma^1, & \gamma^3 &= \sigma^3 \otimes \sigma^3.
\end{aligned} \tag{A.2}
$$

In (A.1) ε refers to the following 2×2 matrix

$$
\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{A.3}
$$

The $\tilde{},\hat{}$ in (A.1) is used to keep track of which 2-component spinor the 2×2 matrices act. Thus we have the following anti-commutation relations

$$
\left\{\Gamma^M, \Gamma^N\right\} = \eta^{MN}, \qquad \{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu}. \tag{A.4}
$$

Note that all the 10 gamma matrices are purely imaginary. We now examine the supersymmetry variations (4.22) with this gamma matrix conventions. Substituting the anstaz in (4.20) in the supersymmetry variations and setting the dilatino variation to zero gives $F_{MNP}\Gamma^{MNP}\epsilon = 0$, expanding this out we obtain

$$
\begin{aligned}
F^{MNP}\Gamma_{MNP} &= \frac{3!}{2!}\left(\Gamma_{\mu\nu}\Gamma_\phi F^{\mu\nu\phi} + \Gamma_{\mu\nu}\Gamma_{\tilde{\phi}} F^{\mu\nu\tilde{\phi}}\right), \\
&= 3\left[-\frac{1}{2}e^{\frac{1}{2}(H+G)}\gamma_{\mu\nu}\Gamma_4 F^{\mu\nu}e^{-(H+G)} - \frac{1}{2}e^{\frac{1}{2}(H-G)}\gamma_{\mu\nu}\Gamma_5 \tilde{F}^{\mu\nu}e^{-(H-G)}\right], \\
&= -\frac{3}{2}\left[\gamma_{\mu\nu}\Gamma_4 F^{\mu\nu} - i\gamma^{\mu\nu}\gamma^5 F_{\mu\nu}\Gamma_5\right]e^{-\frac{1}{2}(H+G)}, \\
&= -\frac{3}{2}\gamma_{\mu\nu}F^{\mu\nu}\Gamma_4\left(1 + \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\right)e^{-\frac{1}{2}(H+G)}.
\end{aligned} \tag{A.5}
$$

In the above manipulations we have used the fact that the 3-form field strength is self-dual in 6-dimensions and

$$\gamma^5 = i\Gamma^0\Gamma^1\Gamma^2\Gamma^3, \qquad \epsilon^{\mu\nu\rho\sigma}\gamma_{\rho\sigma} = -2i\gamma^{\mu\nu}\gamma^5. \tag{A.6}$$

Thus to set the dilatino variation to zero we need the following condition on the 6d chirality condition on the spinor

$$\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\epsilon = -\epsilon. \tag{A.7}$$

Since the spinor ϵ is a 10 Weyl spinor we also have the following condition

$$\Gamma^6\Gamma^7\Gamma^8\Gamma^9\epsilon = -\epsilon. \tag{A.8}$$

We now look at the gravitino variation in the 4 directions and obtain

$$\begin{aligned} \delta\psi_\mu &= \nabla_\mu\epsilon + \frac{1}{48}\frac{3}{2}\gamma_{\rho\sigma}F^{\rho\sigma}e^{-\frac{1}{2}(H+G)}\Gamma_4\left(1+\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\right)\Gamma_\mu\epsilon^*, \\ &= \nabla_\mu\epsilon + \frac{1}{16}\gamma_{\rho\sigma}F^{\rho\sigma}e^{-\frac{1}{2}(H+G)}\Gamma_4\Gamma_\mu\epsilon^*, \\ &= \nabla_\mu\epsilon - \frac{1}{16}\gamma_{\rho\sigma}F^{\rho\sigma}e^{-\frac{1}{2}(H+G)}\gamma_\mu\tilde{\varepsilon}\hat{\sigma}_1\epsilon^*. \end{aligned} \tag{A.9}$$

Note that the coefficients occurring in this equation are real due to our convention of the 10d gamma matrices. Let us now look at the remaining components of the gravitino variation

$$\begin{aligned} \delta\psi_\phi &= \nabla_\phi\epsilon - \frac{1}{16}\gamma_{\rho\sigma}F^{\rho\sigma}\epsilon^*, \\ &= \partial_\phi\epsilon - \frac{1}{4}\partial_\mu(H+G)e^{\frac{H+G}{2}}\Gamma_4\Gamma_\mu\epsilon - \frac{1}{16}\gamma_{\rho\sigma}F^{\rho\sigma}\epsilon^*. \end{aligned} \tag{A.10}$$

Similarly the last component of the gravitino variation becomes

$$\begin{aligned} \delta\psi_{\tilde\phi} &= \nabla_{\tilde\phi}\epsilon - \frac{1}{16}\gamma_{\rho\sigma}F^{\rho\sigma}\Gamma_4\Gamma_5e^{-G}\epsilon^*, \\ &= \partial_{\tilde\phi}\epsilon - \frac{1}{4}\partial_\mu(H-G)e^{\frac{H-G}{2}}\Gamma_5\Gamma_\mu\epsilon - \frac{1}{16}\gamma_{\rho\sigma}F^{\rho\sigma}e^{-G}\Gamma_4\Gamma_5\epsilon^*. \end{aligned} \tag{A.11}$$

Let us define the following linear combinations

$$\begin{aligned} \delta\Omega_H &= \delta\psi_\phi - e^G\Gamma_5\Gamma_4\delta\psi_{\tilde\phi}, \\ \delta\Omega_G &= \delta\psi_\phi + e^G\Gamma_5\Gamma_4\delta\psi_{\tilde\phi}. \end{aligned} \tag{A.12}$$

we obtain the equations

$$\begin{aligned} \delta\Omega_H &= -\frac{1}{2}\partial_\mu H\Gamma_4\Gamma^\mu\epsilon + e^{-\frac{H+G}{2}}\partial_\phi\epsilon - e^{-\frac{H-G}{2}}\Gamma_5\Gamma_4\partial_{\tilde\phi}\epsilon, \\ \delta\Omega_G &= -\frac{1}{2}\partial_\mu G\Gamma_4\Gamma^\mu + e^{-\frac{H+G}{2}}\partial_\phi\epsilon + e^{-\frac{H-G}{2}}\Gamma_5\Gamma_4\partial_{\tilde\phi}\epsilon - \frac{1}{8}\gamma_{\rho\sigma}F^{\rho\sigma}\epsilon^*. \end{aligned} \tag{A.13}$$

We now use the chirality condition (A.7) and the representation of the 10d gamma matrices in (A.1) to write the above equations as

$$\begin{aligned} \delta\Omega_H &= \frac{1}{2}\partial_\mu H\gamma^\mu\tilde{\varepsilon}\hat{\sigma}_1\epsilon + e^{-\frac{H+G}{2}}\partial_\phi\epsilon - ie^{-\frac{H-G}{2}}\gamma^5\partial_{\tilde\phi}\epsilon, \\ \delta\Omega_G &= \frac{1}{2}\partial_\mu G\gamma^\mu\tilde{\varepsilon}\hat{\sigma}_1\epsilon + e^{-\frac{H+G}{2}}\partial_\phi\epsilon + ie^{-\frac{H-G}{2}}\gamma^5\partial_{\tilde\phi}\epsilon - \frac{1}{8}\gamma_{\rho\sigma}F^{\rho\sigma}\epsilon^*. \end{aligned} \tag{A.14}$$

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Note that the coefficients of the above equations are also real, therefore we take the real parts of the equations in (A.10) and (A.14). This removes the complex conjugate operation on the spinor. We then consider the following spinor

$$\tilde{\epsilon} = \epsilon_R + i(1 \otimes \tilde{\varepsilon} \otimes \hat{\sigma_1})\epsilon_R. \tag{A.15}$$

Here the subscripts stand for the real part of the spinor ϵ, note that the 1 refers to the 8×8 identity matrix. The above form of $\tilde{\epsilon}$ is in fact a reality condition on $\tilde{\epsilon}$, note that the reality condition does not involve the 4-d spinor. Thus the 4-d spinor is in general complex. The reality condition explicitly is given by

$$\tilde{\epsilon} + \tilde{\epsilon}^* = i(1 \otimes \tilde{\varepsilon} \otimes \hat{\sigma_1})(\tilde{\epsilon} - \tilde{\epsilon}^*). \tag{A.16}$$

The 6 dimensional chirality condition in (A.7) on $\tilde{\epsilon}$ reduces to the following

$$\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5\tilde{\epsilon} = -\tilde{\epsilon}^*, \tag{A.17}$$

while the chirality condition on the T^4 (A.8) directions remains

$$\Gamma^6\Gamma^7\Gamma^8\Gamma^9\tilde{\epsilon} = -\tilde{\epsilon}. \tag{A.18}$$

We now write down the susy variation equation for $\tilde{\epsilon}$ from the equations (A.10) and (A.14), we get the following equations

$$\begin{aligned}
\delta\tilde{\psi}_\mu &= \nabla_\mu\tilde{\epsilon} + \frac{i}{16}\gamma_{\rho\sigma}F^{\rho\sigma}\gamma_\mu\tilde{\epsilon}, \\
\delta\tilde{\Omega}_H &= -i\frac{1}{2}\partial_\mu H\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\partial_\phi\tilde{\epsilon} - ie^{-\frac{H-G}{2}}\gamma^5\partial_{\tilde{\phi}}\tilde{\epsilon}, \\
\delta\tilde{\Omega}_G &= -i\frac{1}{2}\partial_\mu G\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\partial_\phi\tilde{\epsilon} + ie^{-\frac{H+G}{2}}\gamma^5\partial_{\tilde{\phi}}\tilde{\epsilon} - \frac{1}{8}\gamma_{\rho\sigma}F^{\rho\sigma}\tilde{\epsilon}.
\end{aligned} \tag{A.19}$$

We now can take the Kaluza-Klein anstaz for the spinor $\tilde{\epsilon}$

$$\tilde{\epsilon}(x,\phi,\tilde{\phi}) = \exp\left[-\frac{i}{2}(\eta\phi + \tilde{\eta}\tilde{\phi})\right]\tilde{\epsilon}(x) \tag{A.20}$$

Substituting this ansatz in the equations (A.19) we obtain

$$\begin{aligned}
\delta\tilde{\psi}_\mu &= \nabla_\mu\tilde{\epsilon} + \frac{i}{16}\gamma_{\rho\sigma}F^{\rho\sigma}\gamma_\mu\tilde{\epsilon}, \\
i\delta\tilde{\Omega}_H &= \partial_\mu H\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\eta\tilde{\epsilon} - ie^{-\frac{H-G}{2}}\gamma^5\tilde{\eta}\tilde{\epsilon}, \\
i\delta\tilde{\Omega}_G &= \partial_\mu G\gamma^\mu\tilde{\epsilon} + e^{-\frac{H+G}{2}}\eta\tilde{\epsilon} + ie^{-\frac{H+G}{2}}\gamma^5\tilde{\eta}\tilde{\epsilon} - \frac{i}{4}\gamma_{\rho\sigma}F^{\rho\sigma}\tilde{\epsilon}.
\end{aligned} \tag{A.21}$$

The above supersymmetry variations are identical to that obtained in [13] for the case of $(1,0)$ 6d supergravity. Therefore we can use the methods discussed in [13] to find the Killing spinors. These equations determine the 4 dimensional component of the 10 dimensonal Killing spinor. The remaining components are determined by the the conditions (A.16), (A.17) and (A.18).

A.1 Killing spinors in LLM coordinates

We will now find the Killing spinors for $AdS_3 \times S^3$ in LLM coordinates. In these coordinates the solution is given by [13]

$$ds_6^2 = -h^2(dt + V_i dx^i)^2 + h^2(dy^2 + \delta_{ij} dx^i dy^j) + y(e^G d\Omega_1^2 + e^{-G} d\tilde{\Omega}_1^2), \qquad (A.22)$$

$$F_{(2)} = -2\left[d\left(ye^G\right) \wedge (dt + V) + h^2 e^G *_3 d\left(ye^{-G}\right)\right]. \qquad (A.23)$$

where

$$h^{-2} = 2y\cosh G, \qquad z \equiv \frac{1}{2}\tanh G, \qquad dV = -\frac{1}{y} *_3 dz. \qquad (A.24)$$

and z satisfies the following equations

$$\left(\partial_i^2 + y\partial_y \frac{1}{y}\partial_y\right) z = 0, \qquad (A.25)$$

$$\partial_i z \partial_i z + \partial_y z \partial_y z = \frac{(1-4z^2)^2}{4y^2}.$$

Let us first obtain the 4 dimensional part of the Killing spinor, we will then use the conditions (A.16), (A.17) and (A.18) to obtain the full 10 dimensional Killing spinor. There are two choices for the Killing spinor of the above solution: $\eta = 1, \tilde{\eta} = -1$, or $\eta = -1, \tilde{\eta} = +1$ [13]. We now solve for the Killing spinor which has the condition $\eta = 1, \tilde{\eta} = -1$. Substituting the solution given in (A.22) into the the Killing spinor equation $\delta\tilde{\Omega}_H = 0$ of (A.21) we obtain

$$\left(\sqrt{1+e^{2G}}\gamma^3 + i\gamma^5 e^G + 1\right)\tilde{\epsilon} = 0. \qquad (A.26)$$

Note that the above operator is a projector. To solve for the Killing spinor we first choose the following ansatz

$$\tilde{\epsilon} = \exp(i\delta\gamma^5\gamma^3)\tilde{\epsilon}_1, \qquad \gamma^3\tilde{\epsilon}_1 = -\tilde{\epsilon}_1. \qquad (A.27)$$

Substituting the above ansatz in the Killing spinor equation given in (A.26) we obtain

$$\left(\sqrt{1+e^{2G}}\gamma^3 + i\gamma^5 e^G + 1\right)\left(\cosh\delta + i\sinh\delta\gamma^5\gamma^3\right)\tilde{\epsilon}_1 = 0. \qquad (A.28)$$

Expanding the above equation and equating the real and imaginary parts we obtain the following

$$\tanh\delta = \frac{\sqrt{1+e^{2G}} - 1}{e^G}, \qquad \tanh\delta = \frac{e^G}{\sqrt{1+e^{2G}} + 1}. \qquad (A.29)$$

From these equations we obtain that

$$\sinh 2\delta = \exp G. \qquad (A.30)$$

We now fix the normalization of the spinor $\tilde{\epsilon}_1$: Consider the following spinor bilinears

$$K^\mu = \bar{\tilde{\epsilon}}\gamma^\mu\tilde{\epsilon}, \qquad L^\mu = \bar{\tilde{\epsilon}}\gamma^\mu\gamma^5\tilde{\epsilon}. \qquad (A.31)$$

Let $\tilde\epsilon_1 = \alpha\tilde\epsilon_0$ where $\tilde\epsilon_0^\dagger\tilde\epsilon_0 = 1$. In the coordinate system of the metric in (A.22) we have $K^t = 1, L_y = 1$, therefore we have the conditions

$$K^t = h\bar{\tilde\epsilon}\gamma^0\tilde\epsilon = 1, \qquad L_y = h\bar{\tilde\epsilon}\gamma^3\gamma^5\tilde\epsilon = -1. \tag{A.32}$$

We impose this normalization by first requiring the projector

$$\gamma_0\gamma^3\gamma^5\tilde\epsilon = \tilde\epsilon. \tag{A.33}$$

Now from the construction of $\tilde\epsilon$ in (A.27) we see that the above condition implies that

$$\gamma_0\gamma^5\tilde\epsilon_1 = -\tilde\epsilon_1. \tag{A.34}$$

To fix the normalization constant α we evaluate the following scalar constructed out of spinor bilinears

$$f_2 = i\bar{\tilde\epsilon}\tilde\epsilon. \tag{A.35}$$

Substituting the form of the Killing spinor we obtain

$$\begin{aligned} f_2 &= i\tilde\epsilon_1 \exp(-i\delta\gamma^3\gamma_5)\gamma^0\exp(i\gamma_5\gamma^3)\tilde\epsilon_1, \\ &= \alpha^2\sinh 2\delta = \alpha^2\exp(G). \end{aligned} \tag{A.36}$$

From [13] we have $f_2 = \exp[(H+G)/2]$, therefore we obtain the value of the normalization constant α as

$$\alpha = \exp\left(\frac{H-G}{4}\right). \tag{A.37}$$

Similar manipulations show that this normalization is consistent with $K^t = 1$ and $L_y = 1$ in these cases one obtains the equation

$$h\cosh(2\delta)\alpha^2 = 1. \tag{A.38}$$

The above equation can be easily shown to be true using the solution in (A.22). As part of the consistency requirement it can be shown using simple gamma matrix manipulations that the other components of the vectors K^μ and L_μ vanish. Thus the Killing spinor is given by

$$\begin{aligned} \tilde\epsilon &= \exp\left(\frac{H-G}{2}\right)\exp(i\delta\gamma_5\gamma^3)\tilde\epsilon_0, \\ \gamma^3\tilde\epsilon_0 &= -\tilde\epsilon_0, \qquad\qquad \gamma_0\gamma^5\tilde\epsilon_0 = -\tilde\epsilon_0, \end{aligned} \tag{A.39}$$

or more explicitly the four component spinor is given by

$$\begin{aligned} \tilde\epsilon_4 &= e^{-\frac{i}{2}(\phi-\tilde\phi)}e^{\left(\frac{H-G}{2}\right)}(\cosh\delta - i\gamma^5\sinh\delta)\tilde\epsilon_0, \\ \tilde\epsilon_0 &= \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\i\\0\end{pmatrix}. \end{aligned} \tag{A.40}$$

It can be verified that the spinor in (A.40) also satisfies the first equation in (A.21). Note that there is a degree of freedom in choosing $\tilde{\epsilon}_0$ which corresponds to rotations in the $(1,2)$-plane. The constraints determining $\tilde{\epsilon}_0$ commutes with the rotation matrix $\gamma_1\gamma_2$, therfore we can also consider the following

$$\tilde{\epsilon}_0 \to \exp(\chi\gamma_1\gamma_2)\tilde{\epsilon}_0. \tag{A.41}$$

In (A.40) we have reinstated the dependence on the coordinates $\phi, \tilde{\phi}$ and set the arbitrary constant phase in $\tilde{\epsilon}_0$ to be zero. Performing the same analysis with $\eta = -1, \tilde{\eta} = 1$ we obtain the following four component Killing spinor

$$\begin{aligned} \tilde{\epsilon}'_4 &= e^{\frac{i}{2}(\phi-\tilde{\phi})} e^{(\frac{H-G}{2})}(\cosh\delta - i\gamma^5 \sinh\delta)\tilde{\epsilon}'_0, \\ \tilde{\epsilon}'_0 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ i \end{pmatrix}. \end{aligned} \tag{A.42}$$

Note that for this case compared to the case with $\eta = 1, \tilde{\eta} = -1$ we have $\delta \to -\delta$ and $\gamma^3\epsilon'_0 = \epsilon'_0$ and $\gamma_0\gamma^5\epsilon'_0 = -\epsilon'_0$, since L_y is now normalized to 1. There is again degree of freedom which corresponds to rotations in the $(1,2)$ plane given by

$$\tilde{\epsilon}'_0 \to \exp(\chi\gamma_1\gamma_2)\tilde{\epsilon}'_0. \tag{A.43}$$

With these ingredients the full 10 dimensional 32 component Killing spinor which satisfies the conditions (A.16), (A.17) and (A.18) is given by

$$\tilde{\epsilon} = \frac{1}{\sqrt{2}}\begin{pmatrix} \tilde{\epsilon}'_4 \\ \tilde{\epsilon}_4 \end{pmatrix} \otimes \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ i \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]. \tag{A.44}$$

The 6 dimensional chirality condition (A.17) is satisfied due to the property $\tilde{\epsilon}^*_4 = i\gamma^5\tilde{\epsilon}'_4$. Furthermore the rotation degree of freedom in the $(1,2)$ plane (A.41) and (A.43) has to be such that both $\tilde{\epsilon}_0$ and $\tilde{\epsilon}'_0$ has to be rotated by the same angle χ.

Instead of working with the real part of the 10d spinor in (A.14) we can also work with the its imaginary part and construct the complex spinor $\tilde{\epsilon} = \epsilon_I + i\tilde{\varepsilon}\hat{\sigma}_1\epsilon_I$. Then one obtains the same set of equations as given in (A.21) with F replaced by $-F$. Going through the same procedure of solving for the complex spinor $\tilde{\epsilon}$ we obtain the following Killing spinor

$$\tilde{\epsilon}' = \frac{1}{\sqrt{2}}\begin{pmatrix} \tilde{\epsilon}'^*_4 \\ \tilde{\epsilon}^*_4 \end{pmatrix} \otimes \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ i \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]. \tag{A.45}$$

Note that the 4 component part of the full spinor for this case has is complex conjugate compared the solution in (A.44). This the because on replacing $F \to -F$ in (A.21), ϵ^*_4 and ϵ'^*_4 are solutions. The remaining components of the 32 dimensional spinor remains the same since the construction of $\tilde{\epsilon}$ for this situation is same as the situation when one works only with the real part of ϵ. It is easy to verify that the spinor in (A.45) satisfies all the conditions given in (A.16), (A.17) and (A.18). Thus we have constructed two Killing spinors (A.44) and (A.45) for $AdS_3 \times S^3 \times T^4$.

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B. Gauge transformation

We now proceed to obtain the relevant gauge transformation parameter for the Neveu-Schwarz B-field appearing in the right hand side of the supersymmetry algebra. The relevant parameter is given in equation (2.34) of [23] which is given by

$$\Lambda^{\alpha}_{\mu} = A^{\alpha}_{\mu\rho}\xi^{\rho} - 2i(V^{\alpha}_{+}\bar{\epsilon}_1\Gamma_{\mu}\epsilon_2^{*} + V^{\alpha}_{-}\bar{\epsilon_1}^{*}\Gamma_{\mu}\epsilon_2), \tag{B.1}$$

where ϵ_1, ϵ_2 are any two Killing spinors and $\Lambda^1_{\mu} = \Lambda^{2*}_{\mu}$ and $A^1_{\mu\rho} = A^{2*}_{\mu\rho}$. The Neveu-Schwarz and the Ramond-Ramond 2-form are the real and imaginary components of $A^{\alpha}_{\mu\rho}$. Here we have converted the gamma matrices used in [23] to our convention by setting $(\Gamma_M)_{\rm Schwarz} = \Gamma_M\Gamma_{11}$. $V^{\alpha}_{\pm}$ are related to the axion dilaton background. For constant dilaton backgrounds[6] one has

$$V^1_{-} = V^2_{+}, \qquad \text{and } \; V^1_{+} = V^{2*}_{+}. \tag{B.2}$$

which are all constants. We now substitute the two Killing spinor solutions (A.44) and (A.45) into the expression for the gauge transformation (B.1). For these solutions the 32-component 10 dimension Killing spinor ϵ is related to $\tilde{\epsilon}$ by

$$\epsilon_1 = \frac{1}{2}\left(\tilde{\epsilon} + \tilde{\epsilon}^{*}\right), \qquad \epsilon_2 = \frac{i}{2}\left(\tilde{\epsilon}' + \tilde{\epsilon}'^{*}\right). \tag{B.3}$$

Since ϵ_1 is purely real and ϵ_2 is purely imaginary, the spinor bilinear $\bar{\epsilon}_1\Gamma_{\mu}\epsilon_2^{*}$ is purely imaginary. From (B.1) and the conditions (B.2) it is easy to see that the contribution to the real part of the gauge transformation parameter Λ^{α}_{μ} from the spinor bilinear is proportional to $i\bar{\epsilon}_1\Gamma_{\mu}\epsilon_2$ while the contribution to the imaginary part is given by $\bar{\epsilon}_1\Gamma_{\mu}\epsilon_2$. Thus the gauge transformation parameter relevant for the Neveu-Schwarz B-field is given by $i\bar{\epsilon}_1\Gamma_{\mu}\epsilon_2$.

We now evaluate the spinor bilinear relevant for the gauge transformation parameter and show that it is a constant. We have

$$i(\bar{\epsilon}_1\Gamma_{\mu}\epsilon_2) = \mathrm{Re}\left(\tilde{\epsilon}_4^{\rm T}\gamma_0\gamma_{\mu}\tilde{\epsilon}_4 + \tilde{\epsilon}_4'^{\rm T}\gamma_0\gamma_{\mu}\tilde{\epsilon}_4'\right). \tag{B.4}$$

Here we have used (B.3) and substituted the solutions for ϵ and ϵ' given in (A.44) and (A.45). Let us first evaluate $\mathrm{Re}(\tilde{\epsilon}_4^{\rm T}\gamma_0\gamma_{\mu}\tilde{\epsilon}_4)$ at say $\phi = 0, \tilde{\phi} = 0$.[7] Using the the expression given in (A.40) we find:

$$\begin{aligned}
\tilde{\epsilon}_4^T\gamma_0\gamma_0\tilde{\epsilon}_4 &= \tilde{\epsilon}_4^T\gamma_0\gamma_3\tilde{\epsilon}_4 = 0, \\
\tilde{\epsilon}_4^T\gamma_0\gamma_i\tilde{\epsilon}_4 &= e_i^{\hat{i}}\tilde{\epsilon}_4^T\gamma_0\gamma_{\hat{i}}\tilde{\epsilon}_4, \\
&= e_i^{\hat{i}}\exp\left(\frac{H-G}{2}\right)\tilde{\epsilon}_0^T\gamma_0\gamma_{\hat{i}}(\cosh 2\delta + i\sinh 2\delta\gamma^5\gamma^3)\tilde{\epsilon_0}.
\end{aligned} \tag{B.5}$$

But using the solution in (A.40) one can show that $\tilde{\epsilon}_0^T\gamma_0\gamma_{\hat{i}}\gamma^5\gamma^3\tilde{\epsilon_0} = 0$. We also have

$$\tilde{\epsilon}_0^T\gamma_0\gamma_{\hat{1}}\tilde{\epsilon_0} = 1, \qquad \tilde{\epsilon}_0^T\gamma_0\gamma_{\hat{2}}\tilde{\epsilon_0} = i. \tag{B.6}$$

[6]The near horizon-geometry of the D1-D5 system has a constant dilaton background.

[7]The giant magnon is located at a definite point along these directions.

Therefore

$$
\begin{aligned}
\mathrm{Re}(\tilde{\epsilon}_4^{\mathrm{T}}\gamma_0\gamma_{\hat{1}}\tilde{\epsilon}_4) &= \exp\left(\frac{H-G}{2}\right)\cosh 2\delta = h^{-1}, \\
\mathrm{Re}(\tilde{\epsilon}_4^{\mathrm{T}}\gamma_0\gamma_{\hat{2}}\tilde{\epsilon}_4) &= 0.
\end{aligned}
\tag{B.7}
$$

We then have

$$
\begin{aligned}
\mathrm{Re}(\tilde{\epsilon}_4^{\mathrm{T}}\gamma_0\gamma_1\tilde{\epsilon}_4) &= e_1^{\hat{1}}\mathrm{Re}(\tilde{\epsilon}_4^{\mathrm{T}}\gamma_0\gamma_{\hat{1}}\tilde{\epsilon}_4), \\
&= h \times h^{-1} = 1. \\
\mathrm{Re}(\tilde{\epsilon}_4^{\mathrm{T}}\gamma_0\gamma_2\tilde{\epsilon}_4) &= e_2^{\hat{2}}\mathrm{Re}(\tilde{\epsilon}_4^{\mathrm{T}}\gamma_0\gamma_0\gamma_{\hat{2}}\tilde{\epsilon}_4), \\
&= 0.
\end{aligned}
\tag{B.8}
$$

A similar calculation yields

$$
\mathrm{Re}(\tilde{\epsilon}_4'^{\mathrm{T}}\gamma_0\gamma_1\tilde{\epsilon}_4') = 1, \qquad \mathrm{Re}(\tilde{\epsilon}_4'^{\mathrm{T}}\gamma_0\gamma_2\tilde{\epsilon}_4') = 0. \tag{B.9}
$$

All the remaining components of the above bilinear vanish. Combining (B.8) and (B.9) the relevant gauge parameter is given by

$$
-i\mathrm{Re}(\epsilon_1^{\mathrm{T}}\gamma_0\gamma_1\epsilon_2) = 1, \qquad -\mathrm{iRe}(\epsilon_1^{\mathrm{T}}\gamma_0\gamma_2\epsilon_2) = 0. \tag{B.10}
$$

But since we have the freedom of rotation of the solution in the $(1,2)$ given by (A.41) and (A.43) we can rotate the above gauge parameter to point along any direction in the $(1,2)$ plane. We refer to the gauge parameter as ω_μ and the non-vanishing components are given by

$$
\omega_1 = \cos\chi, \qquad \omega_2 = \sin\chi. \tag{B.11}
$$

where χ is a constant.

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